UNDER REVIEW : IEEE TRANS. AEROSPACE & ELECTRONIC SYSTEMS

Tracking of Multiple Maneuvering Targets in Clutter using Multiple Sensors, IMM and JPDA Coupled Filtering

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Abstract

We consider the problem of tracking multiple maneuvering targets in clutter using switching multiple target motion models. A novel suboptimal filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach, the joint probabilistic data association (JPDA) technique and coupled target state estimation to a Markovian switching system. The algorithm is illustrated via a simulation example involving tracking of two highly maneuvering, at times closely spaced, targets.

Second revision June 24, 2003

Keywords: Target tracking, state estimation, interacting multiple model algorithm, joint probabilistic data association, coupled target state estimation.

This work was supported by the Office of Naval Research under Grant N00014-01-1-0971.

1 Introduction

We consider the problem of tracking multiple maneuvering targets in clutter. This class of problem has received considerable attention in the literature [2],[5],[6],[11]. The switching multiple model approach has been found to be quite effective in modeling highly maneuvering targets [2], [4]-[7]. In this approach various "modes" of target motion are represented by distinct kinematic models, and in a Bayesian framework, the target maneuvers are modeled by switchings among these models controlled by a Markov chain. In the presence of clutter, the measurements at the sensors may not all have originated from the target-of-interest. In this case one has to solve the problem of data association. An effective approach in a Bayesian framework is that of probabilistic data association (PDA) [2], [4] for a single target in clutter and that of joint probabilistic data association (JPDA) [2], [5], [11] for multiple targets in clutter.

It is assumed that the number of targets is known (say N) and that for each target, a track has been formed (initiated) and our objective is that of track maintenance. In [12] such a problem has been considered for a single target using multiple sensors, PDA and switching multiple models. The optimal solution (in the minimum mean-square error sense) to target state estimation given sensor measurements and absence of clutter, requires exponentially increasing (with time) computational complexity; therefore, one has to resort to suboptimal approximations. For the switching multiple model approach, the interacting multiple model (IMM) algorithm of [7] has been found to offer a good compromise between the computational and storage requirements and estimation accuracy [13]. In the presence of clutter, one has to account for measurements of uncertain origin (target or clutter?). Here too, in a Bayesian framework, one has to resort to approximations to reduce the computational complexity, resulting in the PDA filter [2], [5], [1], [12]. In [12] the IMM algorithm has been combined with the PDA filter in a multiple sensor scenario to propose a combined IMM/MSPDAF (interacting multiple model/ multisensor probabilistic data association filter) algorithm. In [2] and [11] multiple targets in clutter (but without using switching multiple models) have been considered using JPDA filter which, unlike the PDA filter, accounts for the interference from other targets. Various versions of IMMJPDA filters for multiple target tracking using switching multiple models may be found in [3], Sec. 6 of [5], [9] and [10]. While [9] and [10] present uncoupled filters (i.e. assume that different target states are mutually independent conditioned on the past measurements), [3] and [5] present IMMJPDA coupled filters where the conditional target state independence assumption is not made. It has been noted in [8] that the IMMJPDA coupled filter equations of [3] and [5] are heuristic. [8] presents an "exact" JPDA coupled filter for non-switching models using the framework of a linear descriptor system.

In this correspondence we extend the approach of [9] which pertains to uncoupled filtering, to IMMJPDA coupled filtering. The only approximations made are those typical for IMM approaches; there are no other heuristics as in [3] and [5]. We use the standard Markovian switching state-space

systems which, as discussed in Remark 4 in Sec. 3, is equivalent to the linear descriptor system framework of [8]. Track initialization (formation) is assumed to have been made for each target. "Standard" assumptions are used for JPDAF ([11], p. 310 of [5]): a measurement can have only one source; among the possibly several validated measurements, at most one of them can be target-originated and the remaining validated measurements are assumed to be due to false alarms or clutter, and are modeled as independently and identically distributed (i.i.d.) with uniform spatial distribution over the entire validation region ("across all targets"). As in [12] and [9] we use sequential updating of the state estimates with sensor measurements. As noted in Remark 1 in Sec. 3, this is a suboptimal approach since joint measurement association events across sensors are not taken into account; for an optimal approach, one should follow [14]-[16] – see Remark 1 for further details.

2 Problem Formulation

Assume that there are total N targets with the target set denoted as $\mathcal{T}_N := \{1, 2, \cdots, N\}$. Assume that the dynamics of each target can be modeled as one of the n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, 2, \cdots, n\}$ and there are total q sensors. For target r ($r \in \mathcal{T}_N$), the event that model i is in effect during the sampling period $(t_{k-1}, t_k]$ will be denoted by $M_k^i(r)$. Although all the targets share a common model set, any two targets may be in different motion status from time to time. For the j-th hypothesized model (mode), the state dynamics and measurements of target r ($r \in \mathcal{T}_N$) are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r), \tag{1}$$

$$z_k^l(r) = h^{j,l}(x_k(r)) + w_k^{j,l}(r) \quad \text{for} \quad l = 1, \cdots, q \tag{2}$$

where $x_k(r)$ is the system state of target r at t_k and of dimension n_x (assuming all targets share a common state space), $z_k^l(r)$ is the (true) measurement vector (i.e. due to target r) from sensor l at t_k and of dimension n_{zl}, $F_{k-1}^j(r)$ and $G_{k-1}^j(r)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ for target r and $h^{j,l}$ is the nonlinear transformation of $x_k(r)$ to $z_k^l(r)$ ($l = 1, 2, \cdots, q$) for model j. The process noise $v_{k-1}^j(r)$ and the measurement noise $w_k^{j,l}(r)$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q_{k-1}^j (same for all targets) and $R_k^{j,l}$ (same for all targets), respectively. At the initial time t_0, the initial conditions for the system state of target r under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}_0^j(r)$ and the known covariance $P_0^j(r)$. The probability of target r in model j at t_0, $\mu_0^j(r) = P\{M_0^j(r)\}$, is also assumed to be known. The switching from model $M_{k-1}^i(r)$ to model $M_k^j(r)$ is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities $p_{ij} = P\{M_k^j(r)|M_{k-1}^i(r)\}$. Henceforth, t_k will be denoted by k.

In coupled state estimation the states of all targets are estimated jointly [5]. To this end define the "global" state

$$x_k := \mathrm{col}\{x_k(1),\, x_k(2),\, \cdots, x_k(N)\} \tag{3}$$

and the corresponding matrices/vectors $J := \mathrm{col}\{j_1,\, j_2,\, \cdots, j_N\}$ where $j_m \in \mathcal{M}_n$ is model j for target m, $F_k^J :=$ block $-$ diag $\left\{F_k^{j_1}(1),\, F_k^{j_2}(2),\, \cdots, F_k^{j_N}(N)\right\}$, $G_k^J :=$ block $-$ diag $\left\{G_k^{j_1}(1),\, G_k^{j_2}(2),\, \cdots,\right.$ $\left.G_k^{j_N}(N)\right\}$, $v_k^J := \mathrm{col}\left\{v_k^{j_1}(1),\, v_k^{j_2}(2),\, \cdots, v_k^{j_N}(N)\right\}$. Then we have the state equation for the N targets as

$$x_k = F_{k-1}^J x_{k-1} + G_{k-1}^J v_{k-1}^J \tag{4}$$

where $E\{v_k^J v_k^{J\prime}\} = Q_k^J :=$ block $-$ diag $\left\{Q_k^{j_1},\, \cdots, Q_k^{j_N}\right\}$. Similarly define the global measurement vector at sensor l as

$$z_k^l := \mathrm{col}\left\{z_k^l(1),\, z_k^l(2),\, \cdots, z_k^l(N)\right\} \tag{5}$$

and the corresponding vectors $h^{J,l}(x_k) := \mathrm{col}\left\{h^{j_1,l}(x_k(1)),\, \cdots, h^{j_N,l}(x_k(N))\right\}$,
$w_k^{J,l} := \mathrm{col}\left\{w_k^{j_1}(1),\, w_k^{j_2}(2),\, \cdots, w_k^{j_N}(N)\right\}$ where $E\{w_k^{J,l} w_k^{J,l\prime}\} = R_k^{J,l} :=$ block $-$ diag $\left\{R_k^{j_1},\, \cdots, R_k^{j_N}\right\}$. Then the measurement equation for N targets at sensor l (assuming no clutter and perfect detections) is given by

$$z_k^l = h^{J,l}(x_k) + w_k^{J,l} \quad \text{for} \quad l = 1, \cdots, q. \tag{6}$$

Define the global mode $M_k^J := \{M_k^{j_1}(1),\, \cdots,\, M_k^{j_N}(N)\}$. The various targets are assumed to evolve independently of each other. Therefore, the transition probability for the global modes are given by

$$p_{IJ} := P\{M_k^{j_1}(1),\, \cdots,\, M_k^{j_N}(N) \,|\, M_{k-1}^{i_1}(1),\, \cdots,\, M_{k-1}^{i_N}(N)\} = \prod_{l=1}^{N} p_{i_l j_l}. \tag{7}$$

Similarly we have

$$\mu_0^J := P\{M_0^{j_1}(1),\, \cdots,\, M_0^{j_N}(N)\} = \prod_{l=1}^{N} \mu_0^{j_l}(l). \tag{8}$$

Regarding the measurements at sensor l, we follow the notations and definitions used used in [9]. At any time k, some measurements may be due to clutter and some due to the target. The measurement set (not yet validated) generated by sensor l at time k is denoted as $Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, \cdots, z_k^{l(m_l)}\}$ where m_l is the number of measurements generated by sensor l at time k. Variable $z_k^{l(i)}$ $(i = 1, \cdots, m_l)$ is the ith measurement within the set. The validated set of measurements of sensor l at time k will be denoted by Y_k^l, containing $\overline{m}_l$ $(\leq m_l)$ measurement vectors. The cumulative set of validated measurements from sensor l up to time k is denoted as $\mathcal{Z}_1^{k(l)} = \{Y_1^l, Y_2^l, \cdots, Y_k^l\}$. The cumulative set of validated measurements from all sensors up to time k is denoted as $\mathcal{Z}_1^k = \{\mathcal{Z}_1^{k(1)}, \mathcal{Z}_1^{k(2)}, \cdots, \mathcal{Z}_1^{k(q)}\}$ where q is the number of sensors.

Assuming there are no unresolved measurements (i.e. measurement associated with two or more targets simultaneously), any measurement therefore is either associated with a target or caused by clutter. Our goal is to find the global state estimate $\hat{x}_{k|k} := E\{x_k|\mathcal{Z}_1^k\}$ and the associated error covariance matrix $P_{k|k} = E\{[x_k - \hat{x}_{k|k}][x_k - \hat{x}_{k|k}]'|\mathcal{Z}_1^k\}$ where x_k' denotes the transpose of x_k. Included in the above formulation is state estimates of individual targets.

3 IMM/JPDA Coupled Filtering Algorithm

We now modify the IMM/JPDA algorithm of [9] to apply to the coupled system (3)-(8); it will be called IMM/JPDACF (CF stands for coupled filter). The approach of [9], in turn, is based on the approaches of [12], [11], [5] and [2]. As in [12] and [9], for convenience, we confine our attention to the case of 2 sensors; however, the algorithm can be easily adapted to the case of arbitrary q sensors. As the IMM/MSPDAF algorithm is well-explained in [12] and Sec. 4.5 of [5], the JPDAF algorithm is well-explained in Sec. 6.2 of [5] and Sec. 9.3 of [2] and the IMM/JPDA filter is given in detail in [9], we will only briefly outline the basic steps in "one cycle" of the IMM/JPDA coupled filter.

Assumed available: Given the state estimate $\hat{x}_{k-1|k-1}^J := E\{x_{k-1}|M_{k-1}^J, \mathcal{Z}_1^{k-1}\}$, the associated covariance $P_{k-1|k-1}^J$ and the conditional mode probability $\mu_{k-1}^J = P[M_{k-1}^J|\mathcal{Z}_1^{k-1}]$ at time $k-1$ for each global mode $J \in \bar{\mathcal{M}}_n := \mathcal{M}_n \times \cdots \times \mathcal{M}_n$.

Step 3.1. Interaction – mixing of the estimate from the previous time ($\forall J \in \bar{\mathcal{M}}_n$):

$$\text{predicted mode probability: } \mu_k^{J-} := P\{M_k^J|\mathcal{Z}_1^{k-1}\} = \sum_I p_{IJ}\mu_{k-1}^I. \tag{9}$$

$$\text{mixing probability: } \mu^{I|J} := P\{M_{k-1}^I|M_k^J, \mathcal{Z}_1^{k-1}\} = p_{IJ}\mu_{k-1}^I/\mu_k^{J-}. \tag{10}$$

$$\text{mixed estimate: } \hat{x}_{k-1|k-1}^{0J} := E\{x_{k-1}|M_k^J, \mathcal{Z}_1^{k-1}\} = \sum_I \hat{x}_{k-1|k-1}^I \mu^{I|J}. \tag{11}$$

covariance $E\{[x_{k-1} - \hat{x}_{k-1|k-1}^{0J}][x_{k-1} - \hat{x}_{k-1|k-1}^{0J}]'|M_k^J, \mathcal{Z}_1^{k-1}\}$ of the mixed estimate:

$$P_{k-1|k-1}^{0J} = \sum_I \{P_{k-1|k-1}^I + [\hat{x}_{k-1|k-1}^I - \hat{x}_{k-1|k-1}^{0J}][\hat{x}_{k-1|k-1}^I - \hat{x}_{k-1|k-1}^{0J}]'\}\mu^{I|J}. \tag{12}$$

Step 3.2. Predicted state and measurements for Sensor 1 ($\forall J \in \bar{\mathcal{M}}_n$):

$$\text{State prediction: } \hat{x}_{k|k-1}^J := E\{x_k|M_k^J, \mathcal{Z}_1^{k-1}\} = F_{k-1}^J \hat{x}_{k-1|k-1}^{0J}. \tag{13}$$

State prediction error covariance $E\{[x_k - \hat{x}_{k|k-1}^J][x_k - \hat{x}_{k|k-1}^J]'|M_k^J, \mathcal{Z}_1^{k-1}\}$:

$$P_{k|k-1}^J = F_{k-1}^J P_{k-1|k-1}^{0J} F_{k-1}^{J'} + G_{k-1}^J Q_{k-1}^J G_{k-1}^{J'}. \tag{14}$$

Using (2) and (13), the global mode-conditioned predicted measurement for sensor 1 is $\hat{z}_k^{J,1} := h^{J,1}(\hat{x}_{k|k-1}^J)$. Using linearized (6), the covariance of the mode-conditioned residual $\nu_k^{J,1(I)} := z_k^{1(I)} - \hat{z}_k^{J,1}$, ($z_k^{1(I)} := \mathrm{col}\{z_k^{1(i_1)}, \cdots, z_k^{1(i_N)}\}$), is given by

$$S_k^{J,1} := E\{\nu_k^{J,1(I)}\nu_k^{j,1(I)\prime}|M_k^J, Z_1^{k-1}\} = H_k^{J,1}P_{k|k-1}^J H_k^{J,1\prime} + R_k^{J,1} \tag{15}$$

where $H_k^{J,1}$ is the first order derivative (Jacobian matrix) of $h^{J,1}(.)$ at $\hat{x}_{k|k-1}^{J(0)}$. Note that (15) assumes that $z_k^{1(i_r)}$ originates from the target r.

Step 3.3. Measurement validation for sensor 1 ($\forall j \in \mathcal{M}_n$): First perform measurement validation for each target r ($r \in \mathcal{T}_N$) separately. For target r, the validation region is taken to be the same for all models, i.e., as the largest of them. Let $S_k^{j,1}(r)$ denote the $n_{z1} \times n_{z1}$ submatrix of $S_k^{J,1}$ including the rows and columns of the latter numbered as $(r-1)n_{z1}+m$, $m = 1, 2, \cdots, r$. That is, $S_k^{j,1}(r)$ is based on the information relevant to target r only. Let $\hat{z}_k^{j_r,1}(r)$ denote the $n_{z1} \times 1$ sub-column of $\hat{z}_k^{J,1}$ including the rows of the latter numbered as $(r-1)n_{z1}+m$, $m = 1, 2, \cdots, r$; that is, $\hat{z}_k^{j_r,1}(r)$ is the mode-conditioned predicted measurement of target r for sensor 1. Let ($|A| = \det(A)$)

$$j_r := \arg\left\{\max_{j\in\mathcal{M}_n} |S_k^{j,1}(r)|\right\}. \tag{16}$$

Then measurement $z_k^{1(i)}$ ($i = 1, 2, \cdots, m_1$) is validated if and only if

$$[z_k^{1(i)} - \hat{z}_k^{j_r,1}(r)]'[S_k^{j_r,1}(r)]^{-1}[z_k^{1(i)} - \hat{z}_k^{j_r,1}(r)] < \gamma \tag{17}$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is $V_k^1(r) := c_{n_{z1}}\gamma^{n_{z1}/2}|S_k^{j_r,1}(r)|^{1/2}$ where n_{z1} is the dimension of the measurement and $c_{n_{z1}}$ is the volume of the unit hypersphere of this dimension. After performing the validation for each target separately, the volume of validation region for the whole target set is approximated by $V_k^1 = \sum_{r=1}^N V_k^1(r)$.

Step 3.4. State estimation with validated measurements from sensor 1 ($\forall J \in \bar{\mathcal{M}}_n$): Given $Z_k^1 := \{z_k^{1(1)}, z_k^{1(2)}, \cdots, z_k^{1(m_1)}\}$, define the set of validated measurement for sensor 1 at time k as $Y_k^1 := \{y_k^{1(1)}, y_k^{1(2)}, \cdots, y_k^{1(\overline{m}_1)}\}$ where $\overline{m}_1$ is total number of validated measurement for sensor 1 at time k. and $y_k^{1(i)} := z_k^{1(l_i)}$ with $1 \le l_1 < l_2 < \cdots < l_{\overline{m}_1} \le m_1$ when $\overline{m}_1 \neq 0$. We now consider joint probabilistic data association across targets following [5] and [2], but for global target state; note that here we consider only sensor 1 (see also Remark 1 later). A marginal association event θ_{ir} is said to be effective at time k when the validated measurement $y_k^{1(i)}$ is associated with (i.e. originates from) target r ($r = 0, \cdots, N$ where $r = 0$ means that the measurement is caused by clutter). Assuming that there are no unresolved measurements, a joint association event Θ is effective when a set of marginal events $\{\theta_{ir}\}$ holds true simultaneously. That is, $\Theta = \cap_{i=1}^{\overline{m}_1}\theta_{ir_i}$ where r_i is the index of the target to which measurement $y_k^{1(i)}$ is associated in the event under consideration. Define the validation matrix (as in [5] and [2])

$$\Omega = [\omega_{ir}] \quad i = 1, \cdots, \overline{m}_1, \quad r = 0, \cdots, N \tag{18}$$

where ω_{ir} =1 if the measurement i lies in the validation gate of target r, else it is zero. A joint association event Θ is represented by the event matrix

$$\widehat{\Omega}(\Theta) = [\widehat{\omega}_{ir}(\Theta)] \quad i = 1, \cdots, \overline{m}_1, \quad r = 0, \cdots, N \tag{19}$$

where $\widehat{\omega}_{ir}(\Theta) = 1$ if $\theta_{ir} \subset \Theta$, =0 otherwise. A feasible association event is one where a measurement can have only one source $(\sum_{r=0}^{N} \widehat{\omega}_{ir}(\Theta) = 1 \quad \forall i)$ and where at most one measurement can originate from a target. The feasible association joint events Θ are mutually exclusive and exhaustive.

Following the definitions in [5] and [2], define the binary measurement association indicator $\tau_i(\Theta) := \sum_{r=1}^{N} \widehat{\omega}_{ir}(\Theta)$, $(i = 1, \cdots, \overline{m}_1)$, to indicate whether the validated measurement $y_k^{1(i)}$ is associated with a target in event Θ. Further, the number of false (unassociated) measurements in event Θ is $\phi(\Theta) = \sum_{i=1}^{\overline{m}_1} [1 - \tau_i(\Theta)]$. We will limit our discussion to nonparametric JPDA [5] and [2]. One can evaluate the likelihood that the global mode is J as

$$\Lambda_k^{J,1} := p[Y_k^1 | M_k^J, Z_1^{k-1}] = \sum_{\Theta} p[Y_k^1 | \Theta, M_k^J, Z_1^{k-1}] P\{\Theta | M_k^J, Z_1^{k-1}\}$$

$$= \sum_{\Theta} p[Y_k^1 | \Theta, M_k^J, Z_1^{k-1}] P\{\Theta\}, \tag{20}$$

where (Sec. 6.2 of [5], Sec. 9.3 of [2])

$$P\{\Theta\} = \frac{\phi(\Theta)!\, \epsilon}{\overline{m}_1!} \prod_{s=1}^{N} (P_D)^{\delta_s(\Theta)} (1 - P_D)^{1-\delta_s(\Theta)}, \quad \delta_s(\Theta) := \sum_{i=0}^{\overline{m}_1} \widehat{\omega}_{is}(\Theta), \tag{21}$$

P_D is the detection probability at sensor 1 (assumed to be the same for all targets) and $\epsilon > 0$ is a "diffuse" prior (for nonparametric modeling of clutter) whose exact value is irrelevant. Unlike [9], we do not assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then we have

$$p[Y_k^1 | \Theta, M_k^J, Z_1^{k-1}] = V_1^{-\phi(\Theta)} p[\tilde{Y}_k^1(\Theta) | M_k^J, Z_1^{k-1}] \tag{22}$$

where $\tilde{Y}_k^1(\Theta) \subset Y_k^1$ is a subset of the validated measurements Y_k^1, consisting of the measurements associated with the targets as specified by Θ. The number of measurements in $\tilde{Y}_k^1(\Theta)$ equal $\overline{m}_1 - \phi(\Theta)$ where $\phi(\Theta)$ is the number of false alarms.

Define a $\overline{m}_1 \times [\overline{m}_1 - \phi(\Theta)]$ matrix $\underline{\widehat{\Omega}}(\Theta)$ as a submatrix of $\widehat{\Omega}(\Theta)$ obtained by deleting the first column and all null columns of $\widehat{\Omega}(\Theta)$. Then for a given Θ, we have a measurement vector $\tilde{Y}_k^1(\Theta)$ of dimension $(\sum_{i=1}^{\overline{m}_1} \tau_i(\Theta)) n_{z1}$ given by

$$\tilde{Y}_k^1(\Theta) = (I_{n_{z1}} \otimes \underline{\widehat{\Omega}}'(\Theta)) \mathrm{col}\{y_k^{1(i)}, i = 1, 2, \cdots, \overline{m}_1\} \tag{23}$$

where we stack up all target-associated validated measurements in Θ in ascending order of targets, I_n is the $n \times n$ identity matrix, and the symbol $\otimes$ denotes the Kronecker product. Define a $[(\overline{m}_1 - \phi(\Theta)) n_{z1}] \times [N n_x]$ matrix $H_k^{J,1}(\Theta)$ as a submatrix of $H_k^{J,1}$ obtained by deleting all i-th block

rows ($n_{z1} \times .$) of $H_k^{J,1}$ for which $\delta_i(\Theta) = 0$. That is, we have modified $H_k^{J,1}$ to keep only the block elements associated with target-associated measurements in Θ. It then follows that the linearized measurement equation for $\tilde{Y}_k^1(\Theta)$ is given by

$$\tilde{Y}_k^1(\Theta) = H_k^{J,1}(\Theta)x_k + w_k^{J,1}. \tag{24}$$

Conditioned on the joint association event Θ and mode J, the "coupled" innovations is given by

$$\nu_k^{J,1}(\Theta) = \begin{cases} \tilde{Y}_k^1(\Theta) - \hat{z}_k^{J,1}(\Theta) & \text{if } \delta_r(\Theta) = 1 \text{ for some } r \in \{1, 2, \cdots, N\}, \\ 0 & \text{otherwise,} \end{cases} \tag{25}$$

where $\hat{z}_k^{J,1}(\Theta)$ is a subvector of $\hat{z}_k^{J,1}$ obtained by deleting all i-th block rows ($n_{z1} \times 1$) of $\hat{z}_k^{J,1}$ for which $\delta_i(\Theta) = 0$. The conditional pdf (probability density function) of the validated measurements $\tilde{Y}_k^1(\Theta)$ given their origins (specified by Θ) and the global target mode J, is given by

$$p[\tilde{Y}_k^1(\Theta)|M_k^J, Z_1^{k-1}] = \mathcal{N}(\tilde{Y}_k^1(\Theta); \hat{z}_k^{J,1}(\Theta), S_k^{J,1}(\Theta)) \tag{26}$$

where

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2} \exp[-\frac{1}{2}(x-y)' P^{-1}(x-y)] \tag{27}$$

$$S_k^{J,1}(\Theta) := E\{\nu_k^{J,1}(\Theta)\nu_k^{J,1}(\Theta)'|M_k^J, Z_1^{k-1}\} = H_k^{J,1}(\Theta) P_{k|k-1}^J H_k^{J,1}(\Theta)' + R_k^{J,1}. \tag{28}$$

The probability of the joint association event Θ given that global mode J is effective from time $k-1$ through k is

$$\beta_k^{J,1}(\Theta) := P\{\Theta|M_k^J, Z_1^{k-1}, Y_k^1\} = \frac{1}{c} p[Y_k^1|\Theta, M_k^J, Z_1^{k-1}] P\{\Theta\} \tag{29}$$

where the first term can be calculated from (22)-(28), the second term from (21), and c is a normalization constant such that $\sum_\Theta P\{\Theta|M_k^J, Z_1^{k-1}, Y_k^1\} = 1$.

Using $\hat{x}_{k|k-1}^J$ (from (13)) and its covariance $P_{k|k-1}^J$ (from (14)), one computes the partial update $\hat{x}_{k|k}^{J,1}$ and its covariance $P_{k|k}^{J,1}$ following the standard PDAF [5], [2], except that the global state is conditioned on Θ, not the marginal events θ_{ir}; details follow.

$$\text{Kalman gain:} \quad W_k^J(\Theta) = P_{k|k-1}^J H_k^{J,1}(\Theta)' [S_k^{J,1}(\Theta)]^{-1}. \tag{30}$$

Partial update of the state estimate: $\hat{x}_{k|k}^{J,1}(\Theta) := E\{x_k|\Theta, M_k^J, Z_1^{k-1}, Y_k^1\}$

$$= \begin{cases} \hat{x}_{k|k-1}^J + W_k^J(\Theta)\nu_k^{J,1}(\Theta) & \text{if } \delta_r(\Theta) \neq 0 \text{ for some } 1 \leq r \leq N \\ \hat{x}_{k|k-1}^J & \text{if } \delta_r(\Theta) = 0 \ \forall r \in \{1, 2, \cdots, N\}. \end{cases} \tag{31}$$

$$\hat{x}_{k|k}^{J,1} := E\{x_k|M_k^J, Z_1^{k-1}, Y_k^1\} = \sum_\Theta \beta_k^{J,1}(\Theta)\hat{x}_{k|k}^{J,1}(\Theta) \tag{32}$$

$$\text{Covariance of } \hat{x}_{k|k}^{J,1}: \; P_{k|k}^{J,1} = P_{k|k-1}^{J} - \sum_{\Theta:\Theta\neq\Theta_0} \beta_k^{J,1}(\Theta) W_k^J(\Theta) S_k^{J,1}(\Theta) W_k^J(\Theta)'$$

$$+ \sum_{\Theta} \beta_k^{J,1}(\Theta) W_k^J(\Theta) \nu_k^{J,1}(\Theta) \nu_k^{J,1}(\Theta)' W_k^J(\Theta)'$$

$$- \left[\sum_{\Theta} \beta_k^{J,1}(\Theta) W_k^J(\Theta) \nu_k^{J,1}(\Theta)\right] \left[\sum_{\Theta} \beta_k^{J,1}(\Theta) W_k^J(\Theta) \nu_k^{J,1}(\Theta)\right]' \qquad (33)$$

where Θ_0 denotes Θ for which $\delta_r(\Theta) = 0 \; \forall r \in \{1, 2, \cdots, N\}$. Eqn. (33) follows in a manner similar to eqn. (3.4.2-10) in [5].

Step 3.5. The mode-conditioned predicted measurements for sensor 2 ($\forall J \in \bar{\mathcal{M}}_n$): The "predicted" measurement for sensor 2 is given by

$$\hat{z}_k^{J,2} := h^{J,2}(\hat{x}_{k|k}^{J,1}). \qquad (34)$$

The covariance of the global mode-conditioned residual $\nu_k^{J,2(I)} := z_k^{2(I)} - \hat{z}_k^{J,2}$ is given by

$$S_k^{J,2} := E\{\nu_k^{J,2(I)} \nu_k^{J,2(I)\prime} | M_k^J, \mathcal{Z}_1^{k-1}, Y_k^1\} = H_k^{J,2} P_{k|k}^{J,1} H_k^{J,2\prime} + R_k^{J,2} \qquad (35)$$

where $H_k^{J,2}$ is the first order derivative (Jacobian matrix) of $h^{J,2}(.)$ at $\hat{x}_{k|k}^{J,1}$.

Step 3.6. Measurement validation for sensor 2: This is similar to Step 3.3 where we replace $S_k^{J,1}$ with $S_k^{J,2}$, $z_k^{1(i)}$ with $z_k^{2(i)}$, m_1 with m_2, $V_k^1(r)$ with $V_k^2(r)$, and V_k^1 with V_k^2. Details are similar to that in Step 3.3, hence are omitted.

Step 3.7. Update with validated measurements for sensor 2 ($\forall J \in \bar{\mathcal{M}}_n$): This step is similar to Step 3.4. Using the validated measurements obtained from Step 3.6 and starting from $\hat{x}_{k|k}^{J,1}$ and $P_{k|k}^{J,1}$, one computes the final updates $\hat{x}_{k|k}^{J}$ and $P_{k|k}^{J}$, and the likelihood

$$\Lambda_k^{J,2} := p[Y_k^2 | M_k^J, Y_k^1, \mathcal{Z}_1^{k-1}]. \qquad (36)$$

The details are similar to that in Step 3.4, hence are omitted.

Step 3.8. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$):

$$\mu_k^J := P[M_k^J | \mathcal{Z}_1^k] = \frac{1}{c} \mu_k^{J-} \Lambda_k^{J,1} \Lambda_k^{J,2} \qquad (37)$$

where c is a normalization constant such that $\sum_J \mu_k^J = 1$. For individual targets we have

$$\mu_k^{j_r}(r) := P[M_k^{j_r}(r) | \mathcal{Z}_1^k] = \sum_{j_1=1}^{n} \cdots \sum_{j_{r-1}=1}^{n} \sum_{j_{r+1}=1}^{n} \cdots \sum_{j_N=1}^{n} \mu_k^{j_1,\cdots,j_{r-1},j_r,j_{r+1},\cdots,j_N} \qquad (38)$$

with $J = (j_1, \cdots, j_N)$ in (37).

Step 3.9. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final global state estimate update at time k is given by $\hat{x}_{k|k} = \sum_J \hat{x}_{k|k}^J \mu_k^J$ and its covariance is given by $P_{k|k} = \sum_J \left\{ P_{k|k}^J + [\hat{x}_{k|k}^J - \hat{x}_{k|k}][\hat{x}_{k|k}^J - \hat{x}_{k|k}]' \right\} \mu_k^J$. The state estimate $\hat{x}_{k|k}(r)$ for target r is the n_x-subvector of $\hat{x}_{k|k}$ consisting of elements $(r-1)n_x + m$, $m = 1, 2, \cdots, n_x$.

Remark 1. In the above algorithm we used sequential updating of the state estimates with measurements (one sensor at a time – see Steps 3.4 and 3.7) as in [9] and [12]. This approach is suboptimal but leads to computational savings as one does not have to simultaneously associate measurements across sensors (as in [14]-[16]). In Step 3.4 we are interested in (an approximation to) $E\{x_k|M_k^J, \mathcal{Z}_1^{k-1}, Y_k^1\}$ which is decomposed as in (32) conditioned on Θ's; measurements Y_k^2 are not considered in this step. If one were to seek $E\{x_k|M_k^J, \mathcal{Z}_1^{k-1}, Y_k^1, Y_k^2\}$, then we would have to follow the approach of [14]-[16] by picking all possible association events across sensors also.

Remark 2. Compared to the uncoupled filtering of [9] where the equations are formulated conditioned on marginal association events θ_{ir}, here we have conditioning on joint association events Θ for coupled filtering. Eqn. (26) does not decompose into the product of marginal probabilities as in [9].

Remark 3. Partition the set of all Θs into disjoint sets $\overline{\Theta}_i$s such that

$$\overline{\Theta}_i := \{\Theta \,|\, \delta_r(\Theta) = \delta_r(\tilde{\Theta})\ \forall r,\ \tilde{\Theta} \in \overline{\Theta}_i\}$$

where $i = 1, 2, \cdots, K$. For instance, for $N = 2$, we have $K = 4$ with $\overline{\Theta}_1$ = all Θs in which there are two validated measurements associated with two targets, $\overline{\Theta}_2$ = all Θs in which one validated measurement is associated with target 1 and none with target 2, $\overline{\Theta}_3$ = all Θs in which one validated measurement is associated with target 2 and none with target 3, and $\overline{\Theta}_4$ = all Θs in which none of the validated measurements are associated with any target. It is then easily seen that $W_k^J(\Theta)$, $H_k^{J,1}(\Theta)$, $S_k^{J,1}(\Theta)$ and $\beta_k^{J,1}(\Theta)$ in Step 3.4, all are invariant for $\Theta \in \Theta_i$. This fact can be used to simplify computations in (31)-(33). Similar comments apply to Step 3.7.

Remark 4. If one substitutes (23) into (24), then one obtains a linear descriptor system type of equation such as (12) in [8]. Therefore, the standard state-space system framework used in this paper and the linear descriptor system framework used in [8] are equivalent (except that [8] uses non-switching models whereas we use Markovian switching models). We have retained the notation and formulation of the earlier papers in the field (e.g. [5], [7], [9] and [12]) whereas [8] prefers to follow a linear descriptor system formulation.

4 Simulation Examples

We now consider tracking two highly maneuvering targets in clutter.

4.1 Example 1

The True Trajectory: Target 1 starts at location [21689 10840 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [−8.3 − 399.9 0] and the target stays at constant altitude with a constant speed of 400m/s. Its trajectory is: a straight line with constant velocity between 0 and 17s, a coordinated turn (0.15 rad/s) with constant acceleration of 60 m/s^2 between 17 and

30s, a straight line with constant velocity between 30 and 55s, a coordinated turn (0.1 rad/s) with constant acceleration of 40 m/s^2 between 55 and 70s, and a straight line with constant velocity between 70 and 87s. Target 2 starts at location [30000 $-$ 3040 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [$-$382 157 0] and the target stays at constant altitude with a constant speed of 413m/s. Its trajectory is: a straight line with constant velocity between 0 and 44s, a coordinated turn (0.075 rad/s) with constant acceleration of 30 m/s^2 between 44 and 59s, and a straight line constant velocity between 59 and 87s.

The Target Motion Models: These are exactly as in [9]. The motion models for the two targets are identical. In each mode the target dynamics are modeled in Cartesian coordinates as $x_k(r) = F(r)x_{k-1}(r) + G(r)v_{k-1}(r)$ where the state of the target is position, velocity and acceleration in each of the 3 Cartesian coordinates (x, y and z). Model 1 for nearly constant velocity model with zero mean perturbation in acceleration; Model 2 for Wiener process acceleration (nearly constant acceleration motion); Model 3 for Wiener process acceleration (model with large acceleration increments, for the onset and termination of maneuvers). The details regarding these models may be found in [9]. The initial model probabilities for two targets are identical: $\mu_0^1 = 0.8$, $\mu_0^2 = 0.1$ and $\mu_0^3 = 0.1$. The mode switching probability matrix for two targets is also identical and is as in [9].

The Sensors: Two sensors (we assume collocation, and time synchronization of observations, etc.) are used to obtain the measurements. The measurements from sensor l for model j are $z_k^l = h^{j,l}(x_k)+w_k^{j,l}$, $l = 1, 2$, reflecting range and azimuth angle for sensor 1 (radar), and azimuth and elevation angles for sensor 2 (infrared). The range, azimuth and elevation transformations, respectively, are given by $r = (x^2+y^2+z^2)^{1/2}$, $\quad a = \tan^{-1}(y/x)$, $\quad e = \tan^{-1}[z/(x^2+y^2)^{1/2}]$. The measurement noise $w_k^{j,l}$ for sensor l is assumed to be zero-mean white Gaussian with known covariances $R^1 = \text{diag}[q_r, q_{a1}] = \text{diag}[400\text{m}^2,\ 49\text{mrad}^2]$ with q_{a1} and q_r denoting the variances for the radar azimuth and range measurement noises, respectively, and $R^2 = \text{diag}[q_{a2}, q_e] = \text{diag}[4\text{mrad}^2,\ 4\text{mrad}^2]$ with q_{a2} and q_e denoting the variances for the infrared sensor azimuth and elevation measurement noises, respectively. Both sensors are assumed to be located at the coordinate system origin. The sampling interval was $T = 1$s and it was assumed that the probability of detection $P_D = 0.997$ for both sensors.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda_1 = 20 \times 10^{-6}$/m mrad for sensor 1 and $\lambda_2 = 2 \times 10^{-4}/\text{mrad}^2$ for sensor 2.

Other Parameters: The gates for setting up the validation regions for both the sensors were based on the threshold $\gamma = 16$ corresponding to a gate probability $P_G = 0.9997$.

Simulation Results: The results were obtained from 1000 Monte Carlo runs. Fig. 1 shows the true trajectory of the two targets and the distance between the two targets as a function of time. The two targets start out far apart, move close to each other from 30 to 45 seconds, and then

move apart again. Fig. 2 shows the results of the proposed IMM/JPDACF based on 993 successful runs (target swapping occurred in 7 out of 1000 runs). Fig. 3 shows the results of the uncoupled IMM/JPDAF of [9] based on 982 successful runs (target swapping occurred in 18 out of 1000 runs). Comparing Figs. 2 and 3 we see that when there is no target swapping, differences between the coupled and uncoupled filters are small. To assess the computational requirements of the two approaches, we computed the CPU time needed to execute one time step (averaged over 10 runs and 87 time steps in each run) in MATLAB 6.5 on a 1 GHz (Mobile) Pentium III operating under Windows XP (professional). The uncoupled IMM/JPDAF needs 0.1063 secs compared to 0.1369 secs required by IMM/JPDACF. Thus with a 29% increase in computational cost, IMM/JPDACF results in a 61% fewer target swappings compared with uncoupled IMM/JPDAF. Finally, neither approach experienced any loss of tracks, only target swapping.

4.2 Example 2

We now consider the same secnario as that in Example 1 except for a linear shift in the y-direction in the trajectory of target 2. Target 2 now starts at $[30000 \ \ -3040 + d \ \ 40]$m with d= -500, -250, 250 or 500m. When $d = 0$, we get Example 1. Different values of d lead to different separations and encounters between the trajectories of the two targets: Fig. 5(a),(b) shows the true trajectory of the two targets for two different values of d (-500m and 500m). For each value of d, results were obtained from 100 Monte Carlo runs. Table 1 shows the number of successful runs (no target swapping) versus d for the two approaches IMM/JPDACF and IMM/JPDAF. Fig. 4(c)-(f) shows the position error versus time for different values of d averaged over only the successful runs. It is seen from Table 1 and Fig. 4 that IMM/JPDACF is either better than IMM/JPDAF (e.g. d=-250m or 0m) or similar to IMM/JPDAF (other values of d: larger separation) in performance.

5 Conclusions

We proposed a novel IMM/JPDA coupled filtering algorithm for state (position, velocity and acceleration) estimation for multiple highly maneuvering targets in clutter. The algorithm was illustrated via a simulation example where with a 29% increase in computational cost, IMM/JPDACF resulted in a 61% fewer target swappings compared with uncoupled IMM/JPDAF; neither approach experienced any loss of tracks.

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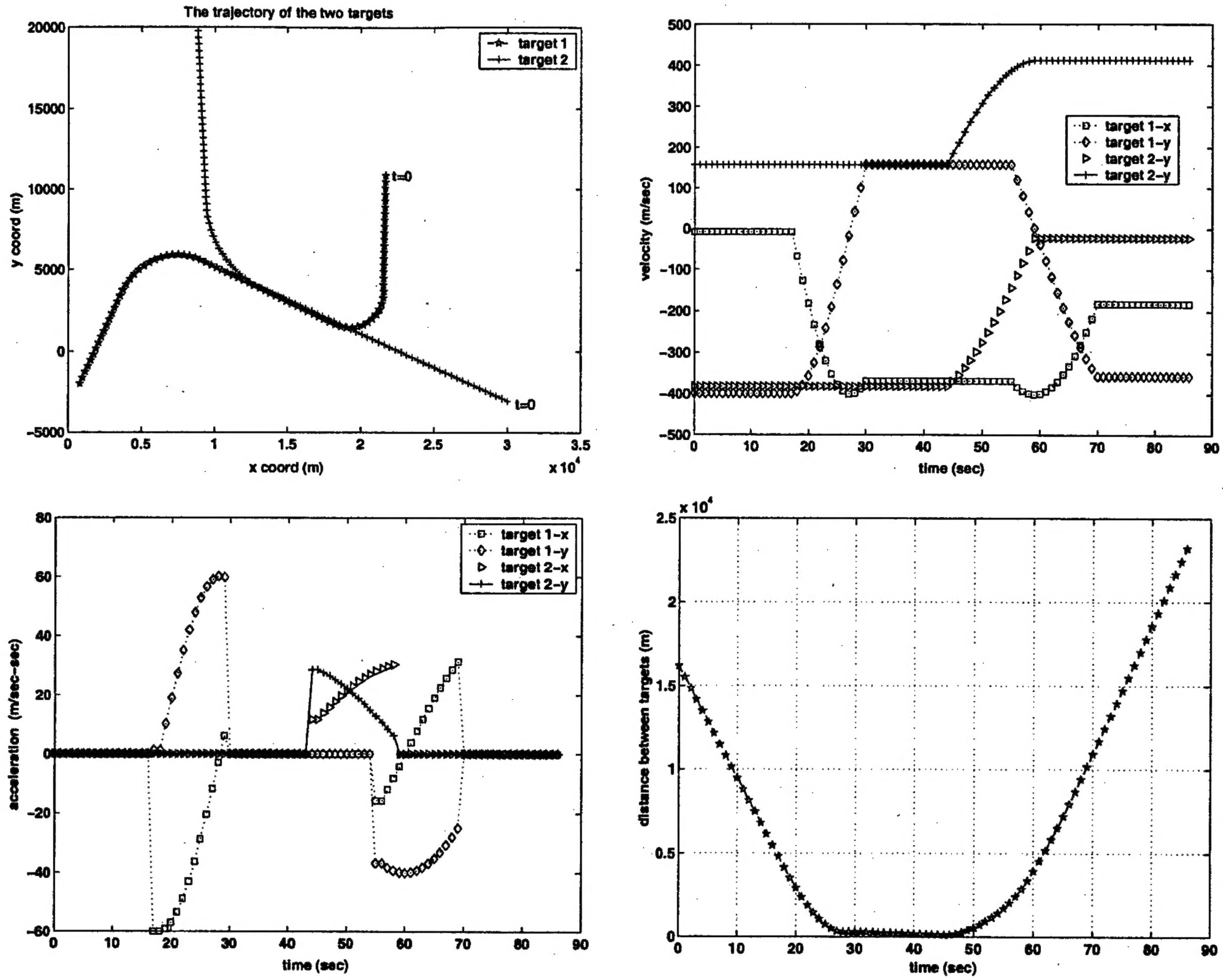

Figure 1: Example 1. The true trajectories of the maneuvering targets (read left-to-right, top-to-bottom): (a) Position in the $xy-$plane. (b) x and y velocities. (c) x and y accelerations. (d) distance between the targets.

d	IMM/JPDACF	IMM/JPDAF
-500m	100/100	100/100
-250m	99/100	96/100
0 m	993/1000	982/1000
250m	100/100	100/100
500m	100/100	100/100

Table 1: *Example 2. Number of successful runs (numerator) out of 100 or 1000 Monte Carlo runs (denominator) for various values of y-shift d.*

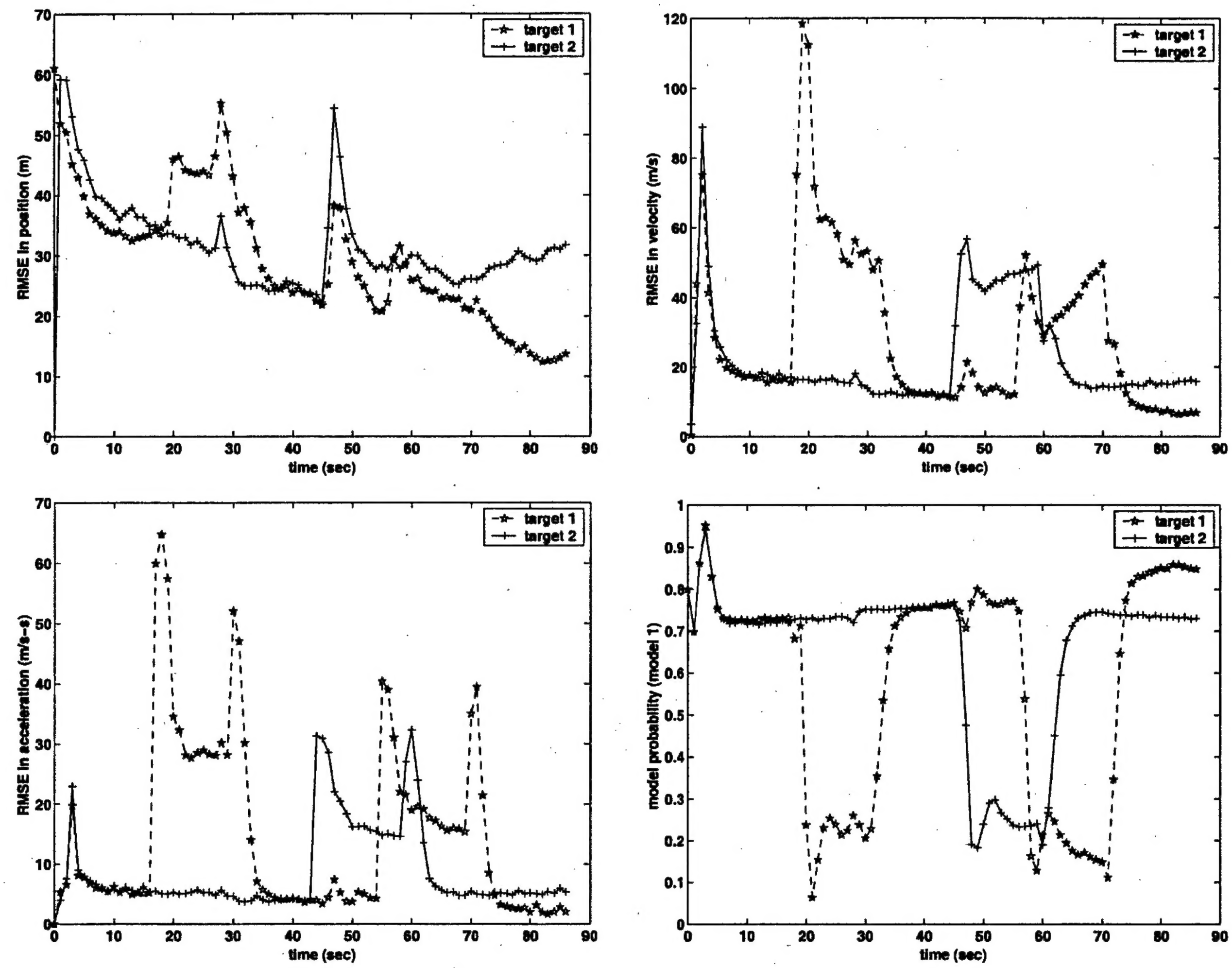

Figure 2: Example 1. Performance of the proposed IMM/JPDACF based on successful (993) runs (read left to right, top to bottom): (a) RMSE in position. (b) RMSE in velocity. (c) RMSE in acceleration. (d) CV model probability $P[M_k^1(r)|\mathcal{Z}_1^k]$ for $r = 1, 2$. (RMSE = root mean-square error; CV = constant velocity)

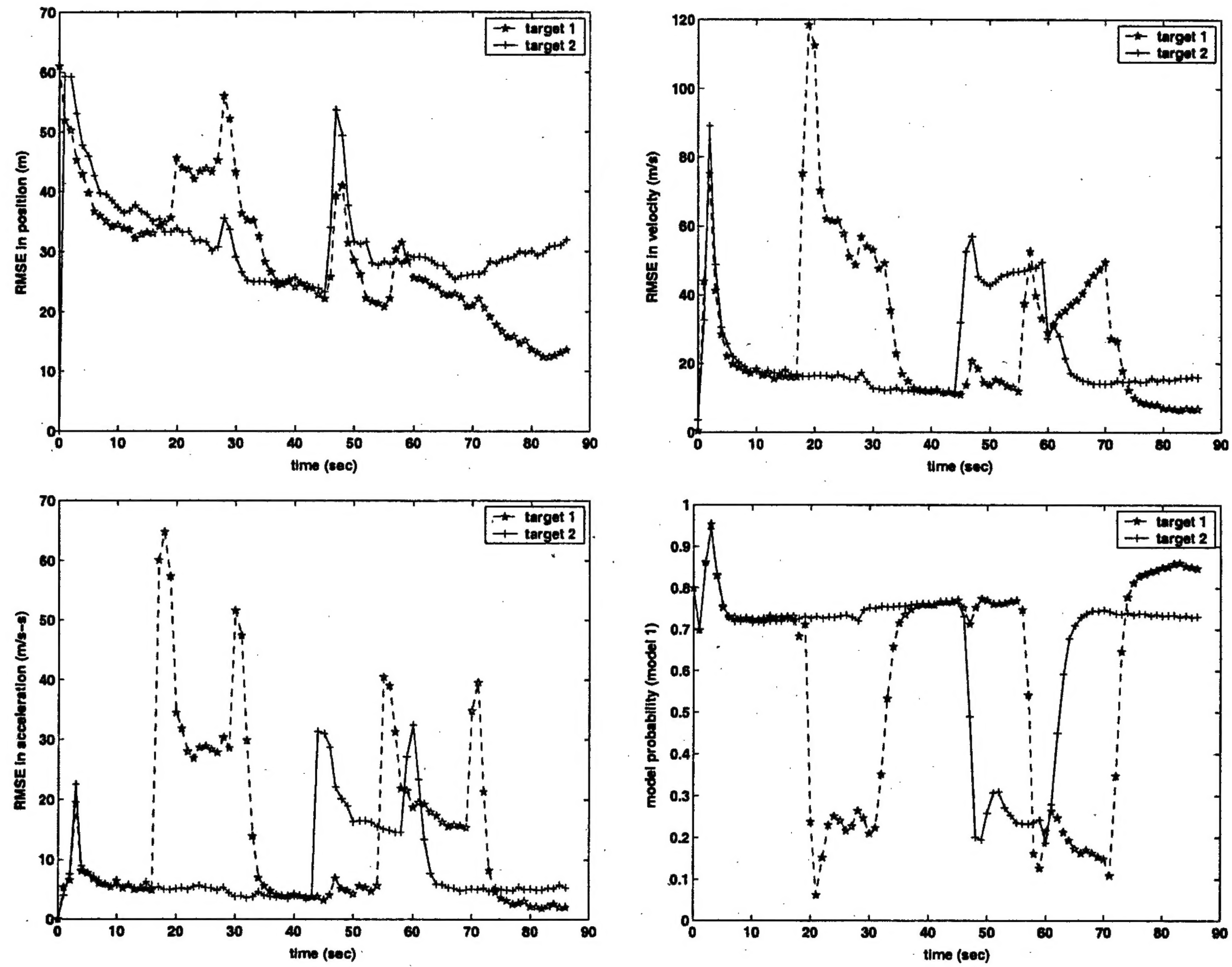

Figure 3: Example 1. Performance of the IMM/JPDAF of [9] based on successful (982) runs (read left to right, top to bottom): (a) RMSE in position. (b) RMSE in velocity. (c) RMSE in acceleration. (d) CV model probability $P[M_k^1(r)|\mathcal{Z}_1^k]$ for $r = 1, 2$. (RMSE = root mean-square error; CV = constant velocity)

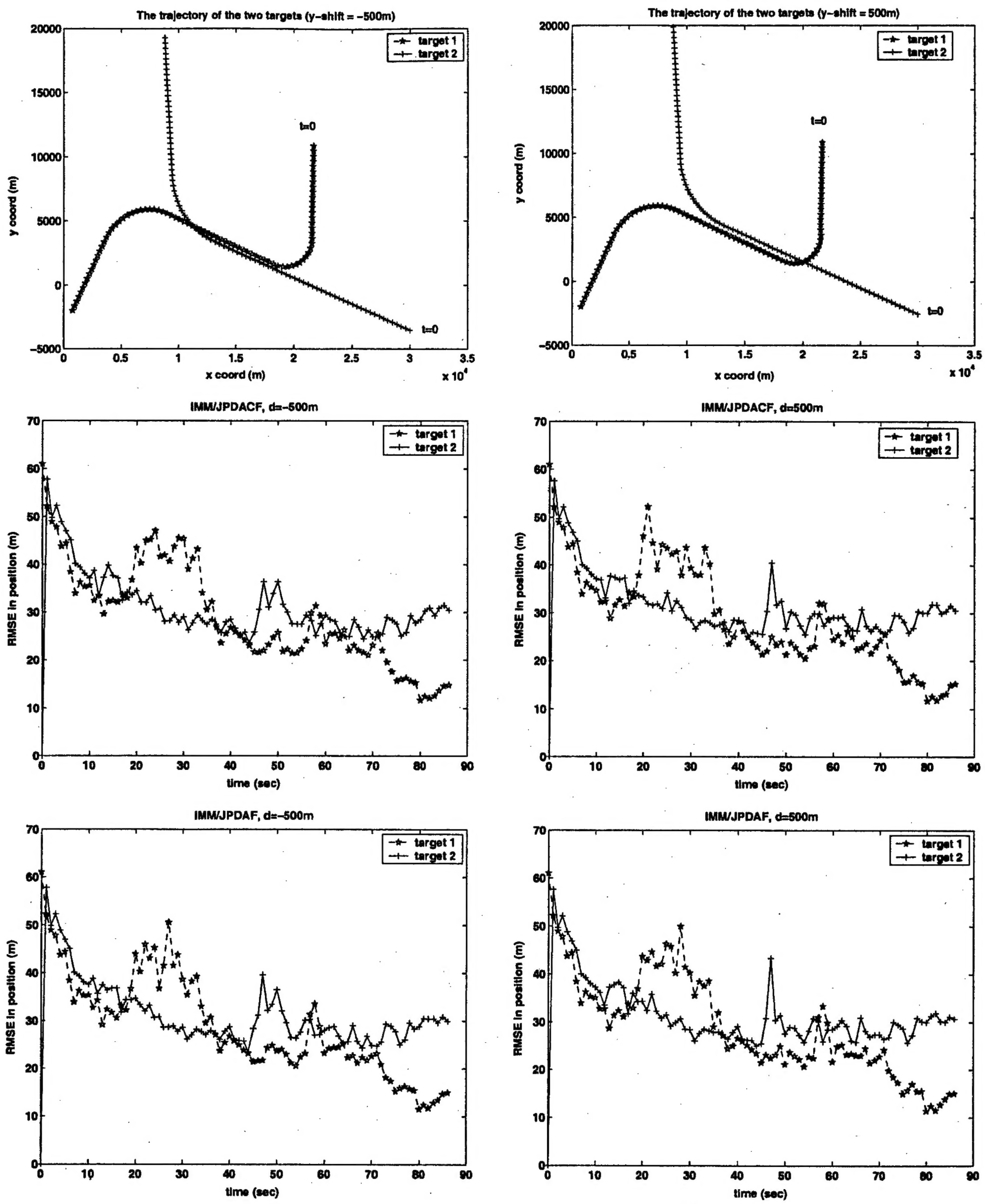

Figure 4: Example 2. The true trajectories of the maneuvering targets in terms of position in the $xy-$plane for two values of y-shift d. (read left-to-right, top-to-bottom): (a) d=-500m, (b) d=500m. Performance (RMSE in position) of the proposed IMM/JPDACF and the IMM/JPDAF of [9] based on successful runs for two values of y-shift d (read left to right, top to bottom): (c) d=-500m, IMM/JPDACF, (d) d=500m, IMM/JPDACF, (e) d=-500m, IMM/JPDAF, (f) d=500m, IMM/JPDAF.